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Alternating-current conductivity of pinned charge-density-wave fluctuations in quasi-one-dimensional conductors

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Abstract. Quasi-one-dimensional conductors which undergo a Peierls transition to a charge-density-wave state at a temperature T_P show a region of one-dimensional fluctuations above T_P. The Ginzburg–Landau–Langevin theory for the frequency-dependent collective conductivity from conductive fluctuations into the charge-density-wave state is developed. By inclusion of a phase-breaking term the effect of local pinning due to random impurities is simulated. It is found that the spectral weight of the unpinned fluctuations is partly redistributed into a pinned mode around a pinning frequency in the far-infrared region. In addition, selection rule breaking by the impurities makes the fluctuating amplitude mode visible in the optical response.

1. Introduction

Quasi-one-dimensional conductors like the transition metal chalcogenides are characterized by a nested Fermi surface. This renders them unstable to the Peierls transition when electron–phonon backscattering between the two sheets of the Fermi surface becomes relevant. We call this a Peierls system (PS). The ensuing charge-density-wave (CDW) which develops below a transition temperature T_P of the order of 200 K shows many unusual properties, especially in its electronic responses [1].

In recent years it became increasingly evident that the electronic properties of PS in their normal phase ($T > T_P$) are also unusual. Photoemission studies (references [2, 3] and earlier work cited therein) point towards possible non-Fermi-liquid (NFL) behaviour. The microwave and optical response also deviates from the Drude predictions for a normal metal [4, 5]. Specifically, the real part of the complex conductivity function $\mathrm{Re}\,\sigma(\omega, T_0)$ for the chain direction and at room temperature T_0 shows the presence of a pseudo-gap at $\omega \approx 2\Delta_0$ where Δ_0 is the zero-temperature half-gap of the corresponding CDW. In addition, a peak structure appears in the far infrared well below the pseudo-gap. This peak resembles the pinned Fröhlich mode at about the same frequency but seen in the fully developed CDW. Similar results were also found for PS films [6].

In principle, these features have been known of for a long time from studies of the CDW material $K_2P_t(CN)_4Br_{0.3}{\cdot}3(H_2O)$ (KCP) in which fluctuation effects due to intrinsic disorder are prominent. In [7] the ac conductivity of KCP is modelled by a dielectric function which takes pinning–unpinning fluctuations into account.

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A related explanation for the optical data is provided by the concept of fluctuating CDW segments which behave similarly to the fully developed CDW. These segments are pinned by random impurities which break the phase invariance of the equation for the fluctuating order parameter.

In view of the one-dimensional nature of fluctuations above the temperature $T^* > T_P$ below which transverse fluctuations set in to initiate the phase transition, the observed effects could also be a signature of NFL behaviour. The latter seems to be established for some of the Bechgaard salts [8–10] where CDW fluctuations are not important.

In the case of PS, the attractive backscattering and the softening of the Peierls phonons make a plain Luttinger-liquid scenario unlikely. Voit [11, 12] advocates a Luther–Emery state [13] for the electrons in blue bronze as an explanation of the photoemission spectra. In this case a spin gap would open before true CDW formation occurred. Such conclusions, however, are not undisputed. Shannon and Joynt [14] argue in favour of a model [15, 16] for a fluctuating Peierls system (FPS). The unusual plasmon dispersion in quasi-one-dimensional conductors can also be understood in terms of a conventional band picture [17]. There seems to be good reason to pursue the FPS concept for one-dimensional metals with CDW fluctuations.

The FPS model has recently been extended by McKenzie [18]. He calculates the one-particle Green's function for Gaussian order parameter fluctuations and large correlation length using Sadovskii's exact method [19]. He points out that the electron spectral function of a FPS is of NFL type as found earlier in the same context [20]. McKenzie also calculates renormalized coefficients for Ginzburg–Landau functionals of PS. The model which we will solve for the collective conductivity corresponds to the FPS concept.

The present paper computes the frequency-dependent conductivity from one-dimensional conductive order parameter fluctuations using a modified, linear Ginzburg–Landau–Langevin (GLL) equation for CDW including phase breaking by impurities. In superconductivity (SC) where phase pinning does not exist, this part of the conductivity was first investigated by Aslamazov and Larkin [21] (AL). The CDW version of the AL theory was given in [22].

From SC it is known [23, 24] that there are two further contributions from order parameter fluctuations to the dc conductivity: a resistive contribution from the reduction of the single-particle density of states which is related to the pseudo-gap and the anomalous Maki–Thompson [25, 26] term which is conductive. In a CDW the latter becomes resistive [27] and dominates the dc conductivity near the transition.

The issue of one-dimensional collective dc conductivity in a CDW was strongly debated in the 1970s [22, 28–32]. In [28] the idea of paraconductivity in one-dimensional metals with dominant electron–phonon interaction was advanced using a GL approach. This result was criticized in [29] where pinned collective fluctuations were shown to reduce the dc conductivity. These authors also studied the corresponding results for the Hubbard and the Tomonaga–Luttinger model. Detailed microscopic studies of FPS in [22, 30] find both resistive and conductive fluctuations but neglect phase pinning. The dc paraconductivity of a commensurate FPS was studied in [32] in a GL context. This paper served as a starting point for the present work.

The paper is organized as follows. Section 2 develops the GLL approach for the CDW and derives the known results for unpinned conductive fluctuations. Section 3 introduces a modified GLL equation where phase invariance is broken in a way which simulates local pinning by random impurities, and evaluates the basic correlation function of the order parameter fluctuations. This result is used to calculate the frequency-dependent collective conductivity exactly within the model. The complicated formula is evaluated approximately for pinning frequencies small in comparison to the frequency of amplitude fluctuations in section 4. Two appendices present mathematical details.

2. The Ginzburg–Landau–Langevin approach

It is interesting to formulate the problem of fluctuation ac conductivity in a CDW without pinning using the GLL method. One expects to find close similarities to the elegant formulations for SC [33, 34]. However, it turns out that one must go beyond overdamped dynamics which in the case of a CDW would only give an instantaneous response in the current correlation function and thus a frequency-independent conductivity.

Our starting point is the GLL equation given directly in terms of the gap fluctuations Δ_k:

$$\ddot{\Delta}_k(t) + \gamma_0 \dot{\Delta}_k(t) + \omega_k^2 \Delta_k(t) = \Gamma_k(t). \tag{1}$$

The parameters γ_0 and ω_k are taken from [28, 31]. Static parameters below correspond to the rigid-lattice values in [18]. For simplicity, the renormalization of the rigid-lattice (RL) values due to fluctuations as proposed in [18] is not considered. Together with Hartree–Fock corrections to the linear GL equation as in [35], it would extend the region of applicability of the linearized approach which is marginal at the RL level. This and the absence of resistive fluctuations prohibit a quantitative comparison with experiments. Below the CDW transition temperature, McKenzie's approach [18] is necessary to give the characteristic optical absorption which has been measured in [36].

The damping constant γ_0 is

$$\gamma_0 = \omega_A^2 \frac{\overline{h}\pi}{8k_B T} \tag{2}$$

where

$$\omega_A^2 = \lambda \omega_Q^2 \tag{3}$$

is the frequency of the amplitude mode of the fully developed CDW [37], ω_Q is the bare frequency of the $2k_F$ phonon which goes soft, and λ is the electron–phonon coupling constant.

The actual frequency ω_k of the amplitude fluctuations in (1) is

$$\omega_k^2 = \omega_0^2(1 + k^2\xi^2) \tag{4}$$

with

$$\omega_0^2 = \omega_A^2 \epsilon_{RL}. \tag{5}$$

For definiteness we assume underdamping ($\gamma_0 < 2\omega_0$) which is the case in the linear fluctuation regime sufficiently above the transition temperature.

Without a form of selection rule breaking, neither ω_A nor ω_0 can be observed in optical conductivity measurements. The amplitude mode ω_A can be seen, however, in Raman scattering [38, 39]. The fluctuating amplitude mode above the transition temperature is observed in neutron scattering studies [40, 41].

The correlation length ξ in (4) is

$$\xi^2 = \xi_0^2/\epsilon_{RL} \tag{6}$$

with ϵ_{RL} given by

$$\epsilon_{RL} = \ln \frac{T}{T_{RL}} \tag{7}$$

where T_{RL} is the rigid-lattice mean-field transition temperature. The reference length ξ_0 is given by [31]

$$\xi_0^2 = \frac{7\zeta(3)\overline{h}^2 v_F^2}{16\pi^2(k_B T)^2}. \tag{8}$$

The complex Gaussian Langevin force Γ has zero mean. Its correlation function

$$\langle \Gamma_k(t)\Gamma^*_{k'}(0)\rangle = \langle |\Delta_k|^2\rangle_0 2\gamma_0\omega_k^2\delta_{k,k'}\delta(t) = 2\gamma_0\omega_0^2\frac{k_BT}{f_0}\delta_{k,k'}\delta(t) \equiv A\delta_{k,k'}\delta(t) \tag{9}$$

is constructed in such a way as to give the fluctuation intensity

$$\langle |\Delta_k|^2\rangle_0 = \frac{k_BT}{a_k}. \tag{10}$$

The latter follows from the linear free-energy functional

$$F_0 = \sum_k a_k|\Delta_k|^2 \tag{11}$$

with

$$a_k = f_0(1+k^2\xi^2) \qquad f_0 = \frac{L\epsilon_{RL}}{\pi\hbar v_F}. \tag{12}$$

Note that (1) implies a spatial correlation of the order parameter according to

$$\langle \Delta(x,0)\Delta^*(0,0)\rangle = \frac{L}{2\pi}\int \mathrm{d}k\ \mathrm{e}^{\mathrm{i}kx}\langle |\Delta_k|^2\rangle_0 = \left(\frac{k_BT\pi\hbar v_F}{2\xi\epsilon_{RL}}\right)\mathrm{e}^{-|x|/\xi} \equiv \psi_{RL}^2\mathrm{e}^{-|x|/\xi}. \tag{13}$$

In the next step the one-dimensional conductivity is computed from the classical Kubo formula

$$\sigma(\omega) = \frac{L}{k_BT}\int_0^\infty \mathrm{d}t\ \mathrm{e}^{\mathrm{i}\omega t}\langle J(t)J(0)\rangle \tag{14}$$

where L is the sample length. The collective current density was calculated in [32]† and reads

$$j(x,t) = \mathrm{i}\frac{b}{2}(\dot\Delta(x,t)\Delta^*(x,t) - \Delta(x,t)\dot\Delta^*(x,t)). \tag{15}$$

The collective current is proportional to the time derivative $\dot\varphi$ of the order parameter phase as for the fully developed CDW, but the prefactor is different. The coefficient b in (15) is [31]

$$b^2 = \left(\frac{e_0}{2k_BT\hbar v_b}\right)^2 \tag{16}$$

involving the backward-scattering rate ν_b due to random static scattering centres. This formula holds in the pure limit when the electron scattering rate obeys

$$\hbar\nu \equiv \hbar(\nu_f + \nu_b/2) < 2\pi k_BT.$$

The homogeneous current density J in (15) is related to j by

$$J(t) = \frac{1}{L}\int_0^L \mathrm{d}x\ j(x,t) = j_{k=0}(t). \tag{17}$$

In the linear setting of (1), not only does Γ obey Gaussian statistics but so also does Δ, and exact Gaussian decoupling gives

$$\langle J(t)J(0)\rangle = \frac{b^2}{2}\sum_k\left[\dot C(k,t)^2 - C(k,t)\ddot C(k,t)\right] \tag{18}$$

provided that the correlation function

$$C(k,t) \equiv \langle \Delta_k(t)\Delta_k^*(0)\rangle \tag{19}$$

† S N Artemenko informed the author that he obtained the same expression for the collective current density by using the Keldysh approach to CDW dynamics.

is real and even in t. $C(k,t)$ is evaluated from (1) and explicitly given by

$$C(k,t) = \langle|\Delta_k|^2\rangle_0 \exp\left(-\frac{\gamma_0}{2}|t|\right)\left[\cos D_k t + \frac{\gamma_0}{2D_k}\sin D_k|t|\right] \tag{20}$$

with

$$D_k = \sqrt{\omega_k^2 - \frac{\gamma_0^2}{4}}. \tag{21}$$

This leads to

$$\dot{C}^2 - C\ddot{C} = \langle|\Delta_k|^2\rangle_0^2\omega_k^2\exp(-\gamma_0|t|). \tag{22}$$

Note that all oscillating terms in the correlation functions cancel out, leaving a purely relaxational response. If one were to use the correlation function

$$C(k,t) = \langle|\Delta_k|^2\rangle_0\exp(-\gamma_k|t|) \tag{23}$$

for the overdamped version of (1) with $\gamma_k = \omega_k^2/\gamma_0$, one would get an instantaneous response $\langle J(t)J(0)\rangle \propto \delta(t)$ and hence a frequency-independent conductivity.

Calculation of the conductivity using (22) gives, however, the correct result given in [35]

$$\mathrm{Re}\,\sigma(\omega) = \sigma_F\frac{\gamma_0^2}{\gamma_0^2+\omega^2} \tag{24}$$

irrespective of the relation between γ_0 and ω_0. The scale value σ_F of the fluctuation conductivity is

$$\sigma_F = \frac{L^2A^4b^2}{16k_BT\omega_0^2\xi\gamma_0^3} \tag{25}$$

and coincides with the result [31]. Explicitly, σ_F reads

$$\sigma_F = \frac{2\pi^2e_0^2k_BTv_F}{\sqrt{7\zeta(3)\epsilon_{RL}}(\hbar v_b)^2}. \tag{26}$$

The conductivity shows the mean-field critical behaviour $\sigma_F \propto \epsilon_{RL}^{-1/2}$. In the picture of a metal with order parameter fluctuations, the collective conductivity adds to the normal-state conductivity

$$\sigma_N = 8e_0^2v_F/(4\pi\hbar v_b).$$

Formal calculations in higher spatial dimensions require a momentum cut-off, in contrast to SC. This is related to the form of the collective current density (15).

3. Breaking of phase invariance

The space-time version of (1) is

$$\ddot{\Delta}(x,t) + \gamma_0\dot{\Delta}(x,t) + \omega_0^2\left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\Delta(x,t) = \Gamma(x,t). \tag{27}$$

The simplest way to break the phase invariance of this equation is to add a pinning term

$$2\omega_i^2|\Delta(x,t)|\cos\varphi(x,t) \tag{28}$$

to the left-hand side, which is a simple local coupling. This is clearly not the general starting point for treating pinning by random impurities [42,43]. However, it will become evident later that this approach simulates local pinning because the final pinning frequency is proportional to the impurity concentration.

To arrive at (28) we start from the more general form

$$\omega_s^2 \sum_i h(x - x_i)(\Delta(x_i, t) + \Delta^*(x_i, t)). \tag{29}$$

The impurities are locally coupled to the order parameter. The real structure function $h(x)$ transmits the effect to the order parameter at x. The terms $\Delta^*(x_i, t)$ break phase invariance by modelling backward scattering. The scale frequencies ω_s and ω_i are different from the final pinning frequency.

We make two further assumptions: the function h is a contact interaction $h(x) = l_i\delta(x)$ with a scattering length l_i. The crudest assumption is, however, that

$$\sum_i \to n_i \int \mathrm{d}x_i \tag{30}$$

where n_i is the density of impurities. This requires $n_i\xi > 1$ and amounts to an early impurity average. Introducing the scale frequency

$$\omega_i^2 \equiv \omega_s^2 n_i l_i \tag{31}$$

then leads to (28).

This admittedly crude model has the advantage of allowing for an exact solution.

The complete GLL equation which replaces (1) can be written as

$$\ddot{\Delta}_k(t) + \gamma_0\dot{\Delta}_k(t) + \omega_k^2\Delta_k(t) + \omega_i^2(\Delta_k(t) + \Delta^*_{-k}(t)) = \Gamma_k(t). \tag{32}$$

The complex order parameter $\Delta(x, t)$ is decomposed into real and imaginary parts:

$$U(x, t) = \mathrm{Re}\,\Delta(x, t) \qquad \text{and} \qquad V(x, t) = \mathrm{Im}\,\Delta(x, t)$$

giving

$$\Delta_k = U_k + \mathrm{i}V_k \tag{33}$$

with complex U_k and V_k which satisfy the usual reality conditions

$$U_{-k} = U_k^* \qquad V_{-k} = V_k^*. \tag{34}$$

The Langevin equation splits into two equations for U_k and V_k:

$$\begin{aligned} &\ddot{U}_k(t) + \gamma_0\dot{U}_k(t) + \omega_k^2 U_k(t) + 2\omega_i^2 U_k(t) = \Gamma_{Uk}(t) \\ &\ddot{V}_k(t) + \gamma_0\dot{V}_k(t) + \omega_k^2 V_k(t) = \Gamma_{Vk}(t). \end{aligned} \tag{35}$$

Here $\Gamma_{Uk}(t)$ and $\Gamma_{Vk}(t)$ are the Fourier transforms of the real and imaginary parts of $\Gamma(x, t)$, respectively. It is possible that the impurities modify the thermal random force $\Gamma(x, t)$. In our model we assume that this is not the case. This assumption is reasonable for $\omega_i \ll \omega_0$. The random forces Γ_{Uk} and Γ_{Vk} are then independent Langevin forces with the same statistical properties as $\Gamma_k(t)$ (cf. (9)) but only half its strength. The two equations (35) become independent and are both isomorphic with (1). However, the frequencies for the U-modes are modified and change their fluctuation intensities:

$$\langle|U_k|^2\rangle = \left(1 + 2\frac{\omega_i^2}{\omega_k^2}\right)^{-1}\frac{k_BT}{2a_k} \qquad \langle|V_k|^2\rangle = \frac{k_BT}{2a_k}. \tag{36}$$

Hence the intensity of the fluctuating order parameter is reduced:

$$\langle|\Delta_k|^2\rangle = \langle|\Delta_k|^2\rangle_0\frac{1 + (\omega_i/\omega_k)^2}{1 + 2(\omega_i/\omega_k)^2}. \tag{37}$$

A thermodynamic derivation for (36) is given in appendix A. In view of the realistic condition $\omega_i \ll \omega_k$, the renormalization of the mean square order parameter is irrelevant.

The order parameter correlation function becomes

$$C(k,t) = p_+ C_+(k,t) + p_- C_-(k,t) \tag{38}$$

with weights

$$p_+ = \frac{1}{2(1+2(\omega_i^2/\omega_k^2))} \qquad p_- = \frac{1}{2} \tag{39}$$

and the replacement

$$D_k \to D_k^{(+)} = \sqrt{\omega_k^2 + 2\omega_i^2 - \frac{\gamma_0^2}{4}} \equiv \sqrt{(\omega_k^{(+)})^2 - \frac{\gamma_0^2}{4}} \tag{40}$$

in the expression (20) for $C(k,t)$ in order to get $C_+(k,t)$ while $C_-(k,t)$ remains unchanged, i.e. formally $D_k^{(-)} = D_k$, $\omega_k^{(-)} = \omega_k$. This solves the GLL equation (32) completely.

4. Discussion of fluctuation conductivity

We define the wavenumber-dependent pinning frequency

$$\omega_p(k) \equiv \frac{\omega_i^2}{D_k}. \tag{41}$$

From (31) it is seen that $\omega_p(k)$ is proportional to the linear impurity concentration. Thus our model simulates local pinning.

Using the condition $\omega_i \ll D_k$, the result of appendix B leads to the following expression for the real part of the fluctuation conductivity:

$$\begin{aligned}\operatorname{Re}\sigma(\omega) = {} & \frac{L}{k_B T}\frac{b^2}{4}\sum_k \langle|\Delta_k|^2\rangle_0^2 \omega_k^2 \\ & \times \left\{ \left[\frac{\gamma_0}{\gamma_0^2+\omega^2}\right]_{AL} + \left[\gamma_0 \frac{\gamma_0^2+\omega_p^2(k)+\omega^2}{(\gamma_0^2+\omega^2+\omega_p^2(k))^2 - 4\omega^2\omega_p^2(k)}\right]_P \right. \\ & \left. + \left[\gamma_0 \frac{\omega_p^2(k)}{4D_k^2}\frac{(3-\gamma_0^2/\omega_k^2)(\gamma_0^2+4D_k^2)-\omega^2}{(\gamma_0^2+\omega^2+4D_k^2)^2 - 16\omega^2 D_k^2}\right]_A \right\}. \end{aligned} \tag{42}$$

Even before the k-summation is performed, three different contributions to the fluctuation conductivity can be distinguished: the relic of the AL conductivity (AL) centred at zero frequency, a pinned mode (P) near the frequency $\omega_p(0)$, and a weak structure (A) associated with the fluctuating amplitude modes ω_k. The latter results from selection rule breaking by the impurities. Thus, traces of the fluctuating amplitude mode should be seen in the optical conductivity. A similar case regarding the pinned Fröhlich mode is found in the fully developed CDW [44, 45].

The k-summation is easily done for the AL part and gives

$$\operatorname{Re}\sigma(\omega)_{AL} = \frac{1}{2}\sigma_F \frac{\gamma_0^2}{\gamma_0^2+\omega^2}. \tag{43}$$

This is exactly half the result (24). The spectral weight W according to

$$W \equiv \int_{-\infty}^{\infty} \mathrm{d}\omega \, \operatorname{Re}\sigma(\omega) \tag{44}$$

is

$$W_{AL} = \frac{\pi}{2}\gamma_0\sigma_F. \tag{45}$$

A lengthy but exact calculation gives for the P mode

$$\mathrm{Re}\,\sigma(\omega)_P = \sigma_F[J_1(\omega) + 2\,\mathrm{Re}\,J_2(\omega)] \tag{46}$$

with

$$J_1(\omega) = \frac{1}{2}\gamma_0^2 \frac{\gamma_0^2 + \omega^2 - 4\omega_i^4/\gamma_0^2}{(\omega^2 + [\gamma_0 + 2\omega_i^2/\gamma_0]^2)(\omega^2 + [\gamma_0 - 2\omega_i^2/\gamma_0]^2)} \tag{47}$$

and

$$J_2(\omega) = 2\frac{\omega_0}{\gamma_0}\frac{\omega_i^4}{4\omega_i^4/\gamma_0^2 - (\gamma_0 + \mathrm{i}\omega)^2}\frac{1}{\sqrt{(4\omega_0^2 - \gamma_0^2)(\gamma_0 + \mathrm{i}\omega)^2 + 4\omega_i^4}}. \tag{48}$$

Note the non-algebraic structure of the conductivity due to J_2. The spectral weight associated with $\mathrm{Re}\,\sigma(\omega)_P$ is independent of pinning parameters and given by

$$W_P = \frac{\pi}{2}\gamma_0\sigma_F. \tag{49}$$

It adds the missing half to the total spectral weight $\pi\gamma_0\sigma_F$ of the unpinned fluctuation conductivity.

The amplitude mode is treated approximately. Assuming $\gamma_0 \ll 2\omega_0$ and retaining the k-dependence only in the prefactor, one finds

$$\mathrm{Re}\,\sigma(\omega)_A = \frac{3}{64}\frac{\omega_i^4}{\omega_0^4}\sigma_F\left\{\frac{\gamma_0^2(12\omega_0^2 - \omega^2)}{(\omega^2 - 4\omega_0^2)^2 + 4\omega^2\gamma_0^2}\right\}. \tag{50}$$

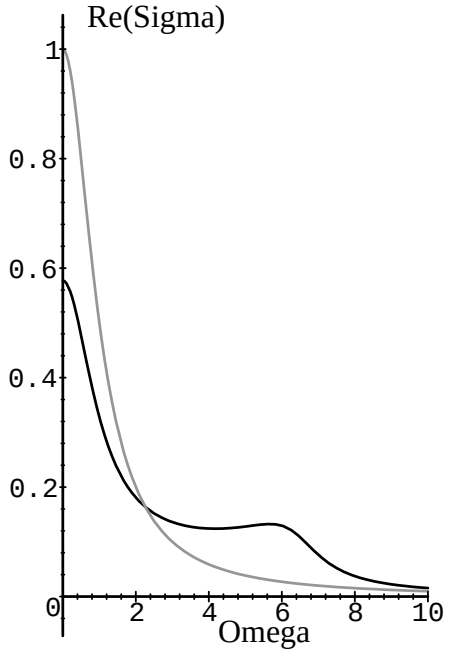

Figure 1. The real part of the scaled fluctuation conductivity (the sum of equation (43) and equation (46)) in comparison to the unpinned case (grey line: equation (24)) as function of the scaled frequency. The conductivity unit is σ_F according to equation (25) and the frequency unit is the damping constant γ_0 (cf. equation (2)) of the fluctuating amplitude mode. In these units the following values were chosen: amplitude mode frequency $\omega_0 = 40$ and pinning scale frequency $\omega_i = 16$. The actual pinning frequency is seen to be near $6\gamma_0$.

The amplitude mode has a peak near $2\omega_0$. Its spectral weight W_A is small and given by

$$W_A = \left(\frac{3}{64}\frac{\omega_i^4}{\omega_0^4}\right)\pi\gamma_0\sigma_F. \tag{51}$$

Figure 1 shows the pinned fluctuation conductivity in comparison with the unpinned case neglecting the weak-amplitude mode.

5. Summary

The Ginzburg–Landau–Langevin method is developed for the fluctuation conductivity in charge-density-wave systems above the transition temperature when fluctuations are one dimensional. An additional phase-breaking term due to impurities is introduced and its consequences for the fluctuation conductivity are evaluated. It is found that the spectral weight of the unpinned fluctuations is partly redistributed into a pinned mode around a pinning frequency in the far infrared, as seen in experiments. In addition, selection rule breaking by the impurities enables traces of the fluctuating amplitude mode to appear in the optical response.

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Appendix A. Pinned fluctuation intensity

The deterministic part of the GLL equation (32) can be expressed in terms of real variables $x_{k\nu}$ as

$$\ddot{x}_{k\nu} + \gamma_0\dot{x}_{k\nu} = -\frac{\omega_k^2}{2a_k}\frac{\partial F}{\partial x_{k\nu}} \tag{A.1}$$

with the energy functional

$$F = \sum_{k,\nu} a_{k\nu}x_{k\nu}^2 \tag{A.2}$$

with

$$a_{k\nu} = \begin{cases} a_k(1+2b(k)) & \nu = 1,2 \\ a_k & \nu = 3,4 \end{cases} \tag{A.3}$$

and

$$b(k) \equiv \frac{\omega_i^2}{\omega_k^2}. \tag{A.4}$$

We have used the decomposition (33) and split the components U_k and V_k into real and imaginary parts:

$$U_k = x_{k1} + \mathrm{i}x_{k2} \qquad V_k = x_{k3} + \mathrm{i}x_{k4}. \tag{A.5}$$

Though in our model the random forces generate no dependence among the $x_{k\nu}$, the latter are not independent since the $x_{k\nu}$ are even under $k \to -k$ for $\nu = 1, 3$ and odd for $\nu = 2, 4$. In terms of independent $x_{k\nu}$, the energy F becomes

$$F = 2\sum_{k>0}\sum_{\nu=1}^{4} a_{k\nu}x_{k\nu}^2 \equiv \sum_{k>0}\sum_{\nu=1}^{4} F_{k\nu}. \tag{A.6}$$

The statistical average of the independent variables is simply

$$\langle x_{k\nu}^2\rangle = \left(\int \mathrm{d}x_{k\nu}\; x_{k\nu}^2 \exp(-\beta F_{k\nu})\right)\Big/\left(\int \mathrm{d}x_{k\nu}\;\exp(-\beta F_{k\nu})\right) \tag{A.7}$$

and gives the results (36).

Appendix B. Calculation of fluctuation conductivity

We use (38) in the Kubo formula

$$\sigma(\omega) = \frac{L}{k_B T}\frac{b^2}{2}\int_0^\infty \mathrm{d}t\; \mathrm{e}^{\mathrm{i}\omega t}\sum_k \left[\dot C(k,t)^2 - C(k,t)\ddot C(k,t)\right]. \tag{B.1}$$

Splitting the correlation function $C(k,t)$ into its constituents gives

$$\dot C^2 - C\ddot C = \sum_{\nu=\pm} p_\nu^2(\dot C_\nu^2 - C_\nu \ddot C_\nu) + p_+p_-\left\{2\dot C_+\dot C_- - C_+\ddot C_- - C_-\ddot C_+\right\}. \tag{B.2}$$

The result (22) translates into

$$\dot C_\nu^2 - C_\nu\ddot C_\nu = \langle|\Delta_k|^2\rangle_0^2(\omega_k^{(\nu)})^2\exp(-\gamma_0|t|) \qquad \nu=\pm. \tag{B.3}$$

Hence

$$\dot C^2 - C\ddot C = \langle|\Delta_k|^2\rangle_0^2\exp(-\gamma_0|t|)\,p_+\left[\omega_k^2+\omega_i^2+\frac{1}{2}\left\{2\dot C_+\dot C_- - C_+\ddot C_- - C_-\ddot C_+\right\}\right]. \tag{B.4}$$

A somewhat tedious calculation gives a formally exact expression for the frequency-dependent fluctuation conductivity:

$$\begin{aligned}\sigma(\omega) = \frac{L}{k_BT}\frac{b^2}{2}\int_0^\infty \mathrm{d}t\;\exp(\mathrm{i}\omega t-\gamma_0 t)\sum_k p_+\langle|\Delta_k|^2\rangle_0^2 \\ \times\left[\omega_k^2+\omega_i^2+\frac{1}{2}\left(\left\{\frac{\omega_k^4+2\omega_k^2\omega_i^2-\gamma_0^2(\omega_k^2+\omega_i^2)/4}{D_k^{(+)}D_k^{(-)}}\right\}\right.\right. \\ \times(\cos(D_k^{(+)}-D_k^{(-)})t-\cos(D_k^{(+)}+D_k^{(-)})t) \\ +(\omega_k^2+\omega_i^2)(\cos(D_k^{(+)}-D_k^{(-)})t+\cos(D_k^{(+)}+D_k^{(-)})t) \\ -\frac{\gamma_0\omega_i^2}{2}\left(\frac{1}{D_k^{(+)}}+\frac{1}{D_k^{(-)}}\right)\sin(D_k^{(+)}-D_k^{(-)})|t| \\ \left.\left.-\frac{\gamma_0\omega_i^2}{2}\left(\frac{1}{D_k^{(+)}}-\frac{1}{D_k^{(-)}}\right)\sin(D_k^{(+)}+D_k^{(-)})|t|\right)_B\right].\end{aligned} \tag{B.5}$$

This result is too complicated to be discussed in full generality. We will take advantage of the fact that in practice the condition $\omega_i^2 \ll D_k^2$ is fulfilled and perform an expansion of (B.5) with respect to

$$\frac{\omega_i^2}{D_k^2} \ll 1. \tag{B.6}$$

This gives

$$\begin{aligned}(\,)_B \to 2(\omega_k^2+\omega_i^2)\cos\frac{\omega_i^2}{D_k}t+\omega_i^4\frac{\omega_k^2-\gamma_0^2/2}{2D_k^4}\cos 2D_kt \\ -\frac{\gamma_0\omega_i^2}{D_k}\sin\frac{\omega_i^2}{D_k}|t|+\frac{\gamma_0\omega_i^4}{2D_k^3}\sin 2D_k|t|-\omega_i^4\frac{\omega_k^2-\gamma_0^2/2}{2D_k^4}\cos\frac{\omega_i^2}{D_k}t.\end{aligned} \tag{B.7}$$

It is easier to do the time integration for the real part of the conductivity in the approximated version of (B.5). The imaginary part follows from the Kramers–Kronig relation. The relevant terms up to order $\omega_p^2(k)$ but neglecting small corrections of numerical constants of order ω_i^2 are given as (42) in section 4.

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